Evolutionary minority game with heterogeneous strategy distribution

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Abstract

We present detailed numerical results for a modified form of the so-called Minority Game, which provides a simplified model of a competitive market. Each agent has a limited set of strategies, and competes to be in a minority. An evolutionary rule for strategy modification is included to mimic simple learning. The results can be understood by considering crowd formation within the population.

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I. INTRODUCTION

There is rapidly growing interest in the study of complex adaptive systems (CAS) [1]. They not only provide a challenging problem for physicists because of the non-trivial self-organizing phenomena which can emerge, but also have potential applications in a variety of economical, biological and financial problems [2–4]. An important step forward in agent-based models of CAS was made by Challet and Zhang [5,6] who proposed the so-called Minority Game (MG) in which an odd number of agents successively compete to be in the minority. The agents make decisions by evaluating the performance of their strategies from past experience and hence they can adapt. The strategies are randomly assigned to the agents in the beginning of the game and are used throughout the game, hence introducing some quenched disorder. The agents have access to global information, which is in turn generated by the actions of the agents themselves. As the game progresses, non-trivial fluctuations arise in the agents' collective decisions - these can be understood in terms of the dynamical formation of crowds consisting of agents using correlated strategies, and anticrowds consisting of agents using the anticorrelated strategies [7]. Challet and coworkers have recently presented a remarkable connection between the MG and spin glass systems [8].

The MG, however, does not incorporate evolution. Agents may get stuck with poorly performing strategies as a result of the initial (random) strategy distribution. Johnson *et al* [9–12] proposed a version of MG which involves an evolving population. In this so-called evolutionary minority game (EMG), all the agents hold one and the same strategy which is simply to follow the most recent trend. Hence the strategy is dynamical. Each agent also carries a probability p characterizing the chance of following the prediction of the strategy. Evolution comes in by allowing agents to modify their p values when their success rate becomes too low, hence mimicking the notion that market participants ought to learn from past mistakes. Surprisingly, agents who either always follow or never follow the trend generally perform better than cautious agents [9].

In an effort to explain the non-trivial behaviour observed in MG and EMG, D'hulst and Rodgers [13] pointed out the relevance of the Hamming distance between strategies and studied a modified version of EMG in which each agent holds a randomly selected strategy and a probability p. The theory gives results which appear to resemble those of EMG. In contrast to EMG, however, each agent now has a fixed (i.e. non-dynamical) strategy throughout the game. Moreover, it has recently been pointed out that this modified model and the basic EMG actually give qualitatively different numerical results [11]. Recently, we presented a theory for the basic EMG which gives good agreement with numerical data [12]. Our theory properly includes the self-interaction of the agents [12].

The modified EMG model of D'hulst and Rodgers [13] is, however, an interesting model in its own right. In particular, it brings together the idea of evolution from the basic EMG and the idea of random initial strategy distribution from the MG. Here we present detailed numerical results for this modified EMG model of D'hulst and Rodgers [13]. The results are contrasted with those for the basic EMG and MG where possible. The paper is organized as follows. The modified EMG model is defined in Sec.II. Section III gives the numerical results. Differences and similarities are pointed out between the modified model and the basic EMG and MG. Our results are summarized in Sec.IV.

II. MODEL

The model consists of an odd number N of agents. Each agent has to choose between two decisions, 0 or 1, at each timestep. The winning side, i.e., the minority side, represents the outcome of the game at that timestep. These outcomes form the global information made known to all agents. As in the MG, the agents make their decision based on the most recent m outcomes. There are a total of 2^m different possible histories. A strategy is defined as a mapping from the history space to the action space. Since for each possible history there are two possible decisions, there exist a total of 2^{2^m} different strategies. Each agent is allowed to pick *one* strategy from the pool of strategies in the beginning of the

game. Some agents may share a common strategy. The strategy remains fixed throughout the game. This random assignment of strategies is identical to that in MG [5], but different from the EMG [9]. As in the EMG, however, evolutionary behaviour is allowed through the assignment of a parameter p to each agent characterizing the probability that the agent follows the prediction of his strategy. The agent thus has a probability $1 - p$ to make the decision opposite to what his strategy predicts. Each agent is randomly assigned a p-value in the beginning of the game. The scores of all agents are set to zero initially. An agent wins and gains one point if he belongs to the minority group. Otherwise, he loses with one point deducted. If the score drops to d $(d < 0)$, an agent is allowed to modify his p value by choosing a value within a range R centered at the original p value and his score is reset to zero. Reflective boundary conditions ensure that p always lies within the range $0 \leq p \leq 1$. Results are found to be insensitive to the particular choice of boundary conditions.

III. RESULTS

Figure 1 shows the distribution of the p-values $P(p)$ among the agents, obtained numerically after the transient stage of the game has died away. The distribution is normalized such that $\int_0^1 P(p)dp = N$. The parameters chosen are $N = 101$, $d = -4$ and $R = 0.2$. The features observed are insensitive to the initial p-distribution. It is observed that $P(p)$ depends sensitively on m, in contrast to the m-independence of $P(p)$ observed in the basic EMG [9,11,14]. For small m, $P(p)$ has peaks near $p \sim 1$ and $p \sim 0$ implying agents who always act according to or opposite to the trend perform better than those with intermediate values of p. This feature is qualitatively similar to that in the basic EMG with the $m = 1$ results closely resembling those reported in Ref. [9]. For small m such that $2 \cdot 2^m \ll N$, the strategies are almost uniformly distributed among the agents and all strategies are played. This distribution of strategies, together with the fact that every agent holds only one strategy, therefore ensures that every strategy and its anticorrelated partner will be used in each turn of the game. The form of $P(p)$ for small m, namely symmetrical about $p = 0.5$ with

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peaks at the extreme values on either side, leads to good cancellation between the actions taken by the agents using anti-correlated pairs of strategies [7]. This self-organization in the population, in turn, has the advantage that it increases the number of winning agents per turn. Although this optimization of a 'global profit' is not the aim of each individual agent when the decision is made, it results from the competition among the agents. As m increases, $P(p)$ gradually flattens off. In the limit of large m, i.e. $2 \cdot 2^m \gg N$, only a small portion of the whole pool of strategies is picked by the agents. It is therefore unlikely that strategies which are anti-correlated to each other are being played. With or without the effect of the p-values, the game is in the random coin-toss limit. There is no advantage to having one particular p-value over another, hence the flat form of $P(p)$ at high m. The inset in Fig. 1 gives the m-dependence of the mean lifespan $L(p)$, which is the average number of turns a certain value p survives between modifications. The features are similar to those in $P(p)$.

The above discussion implies that for small m, the standard deviation (SD) in the number of agents making a certain decision (either 0 or 1) is small due to the cancellation in the actions taken by agents using anti-correlated pairs of strategies. For large m, the SD approaches the random coin-toss limit of $\sqrt{N}/2$. Figure 2 shows the SD as a function of m for $N = 101$. It can be seen that the SD does indeed increase monotonically with m, up to the random coin-toss limit. Also included in Fig. 2 are the results, averaged over different initial distributions of strategies, for the minority game (MG) with $s = 2$. These $s = 2$ MG results are included in order to contrast the different ways in which adaptive behaviour is introduced in the two models. In the $s = 2$ MG, each agent randomly picks two strategies initially, with repetitions allowed. The picked strategies may not always give the opposite action in response to a given history of the most recent m outcomes. In the modified EMG, each agent effectively holds one randomly picked strategy together with its anti-correlated counterpart, and chooses between them stochastically using p. The modified EMG is hence effective in forming similar-sized crowds and anticrowds, thereby yielding a smaller-than-random SD for a wide range of m values (see Fig. 2). The $s = 2$ MG, however, has a much

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higher SD for small m as it does not have the built-in crowd-anticrowd cancellation effect. Also shown in Fig. 2 is the SD for the basic EMG: the SD is m-independent and takes on a value close to the $m = 1$ result of the modified EMG. In the basic EMG, every agent carries the same strategy at a given moment and the self-organized distribution $P(p)$ leads to an effective crowd-anticrowd cancellation which is independent of the size of the strategy space, and hence m.

Figure 3 shows the dependence of $P(p)$ and $L(p)$ (inset) on d. The properly normalized $P(p)$ as shown does not depend on d while $L(p)$ depends on $|d|$ in such a way that $L(p) \sim |d|$. Both features are identical to those in the basic EMG. If we denote $\tau(p)$ as the average winning probability of an agent playing with value p, $L(p) = |d|/(1 - 2\tau(p))$. It should be noted that for all versions of the MG, $\tau < 1/2$. However, the results of $L(p)$ reported here give values of $\tau(p)$ which are quite different from those obtained by numerically solving the set of equations given in Ref. [13]. Figure 4 shows $P(p)$ and $L(p)$ for various values of R. Both $P(p)$ and $L(p)$ show some dependence on R with the peaks on both sides becoming less pronounced as R increases. Comparing with the results of EMG [11], the modified EMG has a more sensitive dependence on R although the results are qualitatively similar. A full explanation of the enhanced sensitivity to R requires us to consider the details of the dynamics of the game, since R controls the effective diffusion in p-space. This will be addressed in future work.

IV. DISCUSSION

We have presented numerical results for a modified EMG. The distribution $P(p)$ is found to be m-dependent, in contrast to the basic EMG. For $m = 1$, the modified EMG gives results similar to the basic EMG. As m increases, the strategy distribution leads to a larger SD than the basic EMG since the crowd-anticrowd cancellation effect gets reduced. For large m, the strategy pool is so large that the game is effectively in the random coin-toss limit. In terms of the behavior of the SD, the present modified EMG interpolates between the basic

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EMG at small m, and the MG at large m. We also found that the R dependences of $P(p)$ and $L(p)$ are enhanced by the implementation of the random strategy distribution.

In both the basic EMG and the modified EMG, the adaptability associated with the stochastic p parameter reduces the SD to values *below* the random coin-toss limit for all values of m. This was reported earlier [9] for the basic EMG: interestingly, another example of reduction of SD to values below the random coin-toss limit was subsequently reported for the so-called Thermal Minority Game (TMG) [15–17]. In the TMG, each agent may use one of his s strategies in each turn according to a probability determined by a parameter T playing the role of 'temperature' [15,17]. It was found that for $s = 2$, for example, the SD for the TMG at small m drops to values below the coin-toss limit. The reason can be understood quantitatively in terms of crowd-anticrowd formation [18]. In essence, the built-in frustration in the basic MG due to the distribution of strategies among agents gives rise to a large SD at small m because of the large size of the crowds as compared to the anticrowds [7]. The introduction of temperature effects helps to build up the crowd-anticrowd cancellation by allowing similar-sized crowds and anticrowds to form [18], hence reducing the SD. The present modified EMG is similar to the TMG in that each agent can be regarded as holding effectively two strategies, together with a stochastic rule for strategy-use at each timestep: one strategy is randomly picked and played with probability p while the anti-correlated strategy is played with probability $1 - p$. Thus, p plays a somewhat similar role to the temperature T in TMG. (Compare Fig. 2 of the present paper with the inset of Fig. 3 in Ref. [16]). However, we emphasize that the ($s = 2$) strategies effectively held by agents in the modified EMG *always* form an anti-correlated pair - this is not the case in TMG. Finally, we note that a variant of the MG was proposed by Challet *et al* in which agents also have $s = 2$ anti-correlated strategies [19]: however the agents did not use a stochastic variable to choose between their strategies at each timestep, hence the results are not directly related to those of the present modified EMG model.

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FIGURES

FIG. 1. The frequency distribution $P(p)$ and average lifespan $L(p)$ (inset) of the modified EMG for $N = 101$, $d = -4$, $R = 0.2$ and different values of m ($m = 1, 4, 7, 10$). In contrast to basic EMG, both $P(p)$ and $L(p)$ depend on m.

FIG. 2. The standard deviation (SD) in the number of agents making a certain decision as a function of m for the modified EMG, the basic EMG, and the MG with $s = 2$. $N = 101$, $d = -4$ and $R = 0.2$.

FIG. 3. The frequency distribution $P(p)$ and average lifespan $L(p)$ (inset) for $N = 101$, $R = 0.2$, $m = 3$ and different values of d ($d = -1, -4, -7, -10$).

FIG. 4. The frequency distribution $P(p)$ and average lifespan $L(p)$ (inset) for $N = 101$, $m = 3$, $d = -4$, and different values of R ($R = 0.5, 1.0, 1.5, 2.0$).